Solutions to exercises

Fisheries Economics and Management

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Comments and corrections are most welcome!

Answer 2.1

a) CPUE is 2010 tonnes per unit of effort (q multiplied with X).
b) The unit of measurement is vessel year. Thus 2010 tonnes is the annual catch of one vessel.

Answer 2.2

a) First, we should find the stock level that produces MSY. The first order condition for maximum of the natural growth function is F'(X)=0. In this case we have F'(X)=r-2rX/K=0 for X=K/2. Substituting for X in the natural growth function gives MSY=F(X=K/2)=rK/2(1-K/2K)=rK/4.
b) The figure should look like in Figure 2.3 panel (b), with K=8 million tonnes and MSY=800 thousand tonnes.
c) You may use the graphical method used in Figure 2.3 panel (b) to derive the sustainable yield curve, H(E). Alternatively, substitute X=H/qE (from the harvest function) for the stock level in *H=F(X)*. This gives H=(rH/qE)(1-rH/qEK). Rearrange this to arrive at H(E)=KqE(1-qE/r).
d) The sustainable yield (equilibrium harvest) is 446.2, 712.9, 707.2 and 435.5 thousand tonnes for 100, 200, 400 and 500 vessels, respectively.

Solution - Exercise 3.1

The parameter values are:
r = 0.25 per year
K = 1000 tonnes
q = 0.05 tonnes per vessel year
p = 1.00 USD per kg
a = 10 000 USD per vessel year

Find (and explain how) equilibrium effort, catch, revenues and costs for each of the following management objectives:
a) Maximise employment in fish harvesting,
This corresponds to maximizing effort: the open-access effort E (see the table below)

b) Maximise harvest to be processed onshore,
This corresponds to the Maximum sustainable yield

c) Maximise resource rent of the fishery,
This is the MEY solution

How could you as the manager of this fishery realise objective *c* given that objective *a* has been followed until now?

There are several ways from Open-access to Maximum economic yields, and both direct and indirect management instruments may be used, including vessel quotas, licenses, resource tax on effort or harvest, and combinations of instruments.

	MSY	∞ (O–A.)	MEY
X	500	200	600
E	2.5	4	2
H	62.5	40	60
TR	62500	40000	60000
TC	25000	40000	20000
π	37500	0	40000

Solution - Exercise 3.2

Demand (marginal value)

$m_A = 1000 - 0.015\ X_A$

$m_B = 1200 - 0.01\ X_B$

1) $m_A\ (X_A = 20\ 000) = 1000 - 0.015 \cdot 20\ 000 = 700$

$m_B\ (X_B = 30\ 000) = 1200 - 0.01 \cdot 30\ 000 = 900$

2) $m^* = m_A = m_B$, and

$X = X_A + X_B\ (\Rightarrow\ X_B = X - X_A)$

$1000 - 0.015X_A = 1200 - 0.010\ (50\ 000 - X_A)$

Solve with respect to X_A

$X_A = 12\ 000$

$X_B = 50\ 000 - 12\ 000 = 38\ 000$

$m^* = 1000 - 0.015 \cdot 12\ 000 = 820$

3) Firm *A* is selling 8000 tonnes

Firm *B* is buying 8000 tonnes

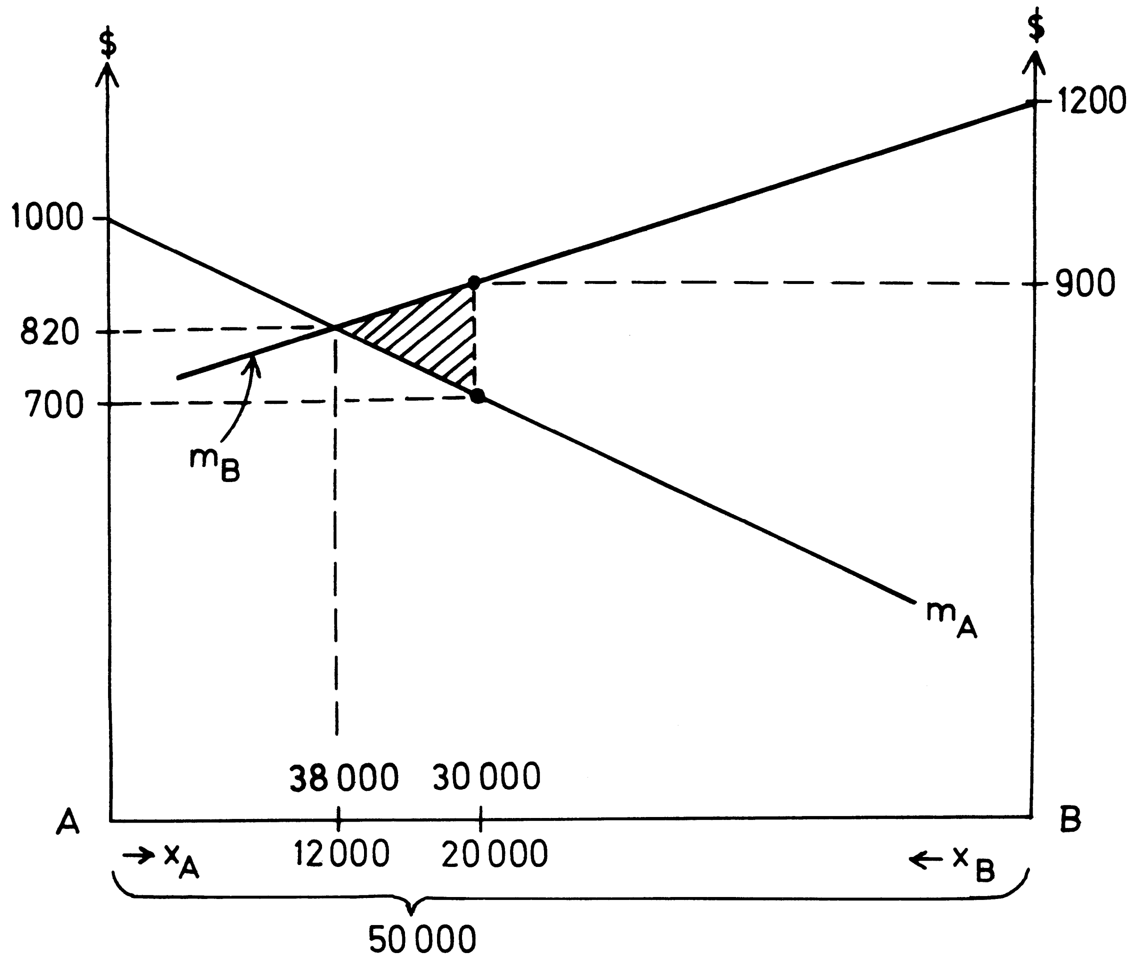
5) Efficiency gain from trade

$$\frac{(900-700)\cdot 8000}{2} = 800\,000\,\$$$

Value of quota: $820 \cdot 50\,000 = 41\,000\,000$ \$

$$\text{Eff. gain in \%: } \frac{800\,000}{41\,000\,000} \cdot 100 \approx 2\%$$

4) Figure



Solution - Exercise 3.3

$$h(E) = a \cdot E - b \cdot E^2$$

a,b positive constants

$TC(E) = c \cdot E$

p = constant price of fish

a) Open-access equilibrium

AR(E) = MC(E) (see equation (3.6))

$p(a - bE) = c$

$pa - c - pbE = 0$

$$E_\infty = \frac{pa - c}{pb} = \frac{a - \frac{c}{p}}{b}$$

$ph = cE$,

$$h = \frac{c}{p} \cdot E\,, \quad h_\infty = \frac{c}{p} E_\infty = \frac{c}{p}\left(\frac{a - \frac{c}{p}}{b}\right)$$

b) $\pi(E) = p(aE - bE^2) - cE$

$$\frac{d\pi(E)}{dE} = pa - c - 2pbE = 0$$

$$E_{\text{MEY}} = \frac{pa - c}{2pb}$$

$$h_{\text{MEY}} = a \cdot E_{\text{MEY}} - b(E_{\text{MEY}})^2 = \frac{pa - c}{4pc}\left[a + \frac{c}{p}\right]$$

c) $E_{\text{MSY}} = \dfrac{a}{2b}$

d) $a = 30$, $b = 0.02$, $c = 100$, $p = 10$

$E_\infty = 1000$, $E_{\text{MSY}} = 750$ (Biological overfishing)

$E_{\text{MEY}} = 500$

e) $t_E = 100$

$TC_P(E) = (c + t_E)E = 200 \cdot E$

(Tax and open access) → $E_\infty = 500 < E_{\text{MSY}} = 750$ (Not biological overfishing)

Exercise 4.1 Answers

Q1: In fishery A the PV of Π^2 is 11.5/1.07 = 10.75 which is less than Π^1.
In fishery B the PV of Π^2 is 1.05/1.07 = 0.98 which is greater than Π^1.
I would recommend a closure of fishery B, but not A.

Q2: In fishery A the social rate of discount has to be less than i given by $11.0 = 11.5/i \Rightarrow i =$ 4.5%,
to make it worthwhile closing also this fishery.

Exercise 4.2 Answers

Q1: $PV = \int_0^\infty A\, e^{-\delta t} dt = A \int_0^\infty e^{-\delta t} dt = A \Big|_0^\infty -\frac{1}{\delta} e^{-\delta t} = \frac{A}{\delta}$.

Q2: We use the formula in Question 1 to find

PV = 10/0.5 = 200 million USD for δ = 5% p.a., and
PV = 10/0.10 = 100 million USD for δ = 10% p.a.

Answers Exercise 5.1

A fish stock X has the following natural growth function

$$F(X) = rX\left(1 - \frac{X}{K}\right) \tag{1.1}$$

Assume that $F(X)$ is the annual natural growth when the size of the stock at the beginning of the year is X.

1. Draw the graph based on (1.1) when r = 0.30 and K = 8000. K is measured in thousand tonnes.

((The $F(X)$-curve runs through X=0 and X=8000 and has its maximum for X=4000, all figures in thousand tonnes. The maximum value equals 600 thousand tonnes.

2. What unit of measure does r have? Discuss the biological parameters r and K using the graph in question 1.

((r is the intrinsic growth rate, and its denomination is “per unit of time”; for instance, “per year”. K is the carrying capacity of the fish stock’s environment. Natural growth increases with r and K)).

3. Assume that no fishing takes place. What is the equilibrium size of the fish stock, according to equation (1.1)?

((The equilibrium size is K))

We introduce the following harvest function

$$H(E, X) = qEX \ , \qquad (1.2)$$

where q is the catchability/availability parameter/coefficient and E is fishing effort.

4. Discuss the catchability parameter q.

((Catch per unit of effort, CPUE, increases in proportion to q for a given stock level)).

With harvest, H, change in the stock level per unit of time is

$$\dot{X} = F(X) - H(E, X) \qquad (1.3)$$

5. Define equilibrium fishing, using function (1.3), and show that the equilibrium harvest, H, can be presented as a function of X. Compare this function with function (1.1). What characterises equilibrium fishing?

((Equilibrium fishing takes place if $dX/dt=0$ for a positive E. In this case equation (1.3) shows that $H=F(X)$. The $F(X)$ function is given in equation (1.1). Harvest equals the natural growth of the stock. The equilibrium stock level is a decreasing function of E, descending from X=K for E=0 to X=0 for a sufficiently large effort.)).

6. Find an expression for the stock level (X_{MSY}) that gives maximum sustainable yield H_{MSY}

(Hint: $\frac{dH}{dX} = 0$ is a necessary condition).

((Taking the derivative of (1.1) and following the hint, gives $X_{MSY}=K/2$. Using this result and substituting for X in (1.1) gives the maximum sustainable yield, $H_{MSY}=rK/4$)).

7. What is the size of X_{MSY} and H_{MSY}, in thousand tonnes and thousand tonnes per year, respectively?

((X_{MSY}=4000 thousand tonnes and H_{MSY}=600 thousand tonnes per year)).

8. Assume that no fishing has taken place and the fish stock is at its pristine/virgin equilibrium. What is the size of the harvest in year 1 when fishing effort is E=100, and $q = 0{,}001 \frac{1}{vessels \cdot year}$?

((800 thousand tonnes)).

9. Explain why the harvest in year 1 (see question 8) is higher than the maximum sustainable harvest/yield you found in question 7.

((The initial stock level, equal to K, is higher than X_{MSY} and the combination of this virgin stock level and the effort (E=100) is sufficient to give a larger harvest than H_{MSY})).

10. Use equations (1.1), (1.2) and (1.3) to find the equilibrium harvest H as a function of effort E (Hint: from (1.2) follows $X=H/qE$).

((Substituting for X from (1.2) into (1.1) gives $H = \frac{rH}{qE}\left(1 - \frac{H}{qKE}\right)$. Solving this with respect to H gives $H = H(E) = KqE\left(1 - \frac{qE}{r}\right)$, (when $H \equiv F(X)$).))

11. What is the equilibrium harvest when fishing effort is kept constant at 100 vessels per year?

((533.3 thousand tonnes))

12. What is the equation for annual total revenue as a function of effort, *TR(E)*, (for equilibrium harvesting) when the price of fish is constant?

((Use what you found in question 10, to arrive at $TR(E) = pH(E) = pKqE\left(1 - \frac{qE}{r}\right)$)).

13. What is the expression for sustainable resource rent when the total cost of the fishery is

$$TC(E) = aE \qquad (1.4)$$

((Using what we found in question 10 and equation (1.4) leads to

$\pi(E) = TR(E) - TC(E) = pKqE\left(1 - \frac{qE}{r}\right) - aE$)).

14. The economic parameters are p = 1.0 \$/kg and a = 1.0 million \$/(vessel·year). What is the size of the equilibrium fish stock in an Open Access fishery? What is the total harvest in this case, and how many vessels participate?

((In general, Open-Access equilibrium requires $MC(E)=AR(E)$. In this particular case with linear functions of total cost and harvest, it easy to see that $TC(E)=TR(E)$. From this follows $pqEX=aE$, which gives the open access equilibrium stock level $X_\infty=a/pq$. For the given parameter values X_∞=1.0 million tonnes.

See question 5 to find $H_\infty=r X_\infty(1- X_\infty/K)$. For the given parameter values H_∞=262.5 thousand tonnes.

Using the harvest function (1.2) and the expressions found for X_∞ and H_∞, we find

$E_\infty = \frac{r}{q}\left(1-\frac{a}{pqK}\right)$. For the given parameter values E_∞=262.5 vessels))

15. What are the optimal/*MEY* fishing effort and the corresponding stock level and harvest? What is the maximum annual resource rent (total and per vessel)?

((The necessary condition for maximisation of resource rent is $MC(E)=MR(E)$. Using this and the expression for $\pi(E)$ found in question 13 gives $E_{MEY} = \frac{r}{2q}\left(1-\frac{a}{pqK}\right)$. Note that by combining this with the results found in question 14, we have the following relationship between the open-access and the maximum resource rent effort: $E_{MEY} = \frac{1}{2}E_\infty$. Using the expression for $\pi(E)$ derived in question 13 and the given parameter values, we get E_{MEY}=131.25 vessels and $\pi(E) = 459.4$ million \$. Resource rent per vessel per year is)).

Solution Exercise 5.2

1. Show that for the Schaefer model the long-run optimal stock level X^* is given by:

$$(*)\qquad X^* = \frac{K}{4}\left[\left(\frac{a}{pqK}+1-\frac{\delta}{r}\right)+\sqrt{\left(\frac{a}{pqK}+1-\frac{\delta}{r}\right)^2+\frac{8a\delta}{pqKr}}\;\right].$$

Solution

Equation (4.18) gives the necessary condition for the optimum, and X* can be derived from this equation

$$(4.18)\quad (p-c(X))F'(X)-c'(X)F(X)=\delta(p-c(X)),$$

by use of the following Schaefer model equations:

(1) $F(X) = rX(1 - X/K)$, and $F'(X) = r - 2rX/K$

(2) $H = qEX$

(3) $TC(X)=aE$

(4) $c(X) = \frac{TC(E)}{H} = \frac{aE}{qEX} = \frac{a}{qX}$, and $c'(X) = -a/qX^2$.

Inserting for *c*(*X*), *c'*(*X*), *F*(*X*) and *F'*(*X*) from equations (1) and (4) into (4.18) gives

(5) $$\left(p - \frac{a}{qX}\right)\left(r - \frac{2rX}{K}\right) - \left(-\frac{a}{qX^2}\right) rX\left(1 - \frac{X}{K}\right) = \delta\left(p - \frac{a}{qX}\right).$$

(6) $$pr - \frac{2pr}{K}X - \frac{ra}{qX} + \frac{2ar}{qK} + \frac{ar}{qX} - \frac{ar}{qK} = \delta p - \frac{\delta a}{qX}.$$

Dividing equation (6) by *X* gives

(7) $$prX - \frac{2prX^2}{K} - \frac{ra}{q} + \frac{2ar}{qK}X + \frac{ar}{q} - \frac{ar}{qK}X - \delta\, pX + \frac{\delta a}{q} = 0.$$

Dividing equation (7) by *pr* and rearranging a little gives:

(8) $$-\frac{2}{K}X^2 + \left(\frac{a}{pqK} + 1 - \frac{\delta}{r}\right)X + \frac{\delta a}{pqr} = 0.$$

The roots of the general quadratic equation

(9) $$AX^2 + BX + C = 0$$

are

(10) $$X_{1,2} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}.$$

Using (10) to find the positive solution of (8) gives

(11) $$X = \frac{K}{4}\left[\left(\frac{a}{pqK} + 1 - \frac{\delta}{r}\right) + \sqrt{\left(\frac{a}{pqK} + 1 - \frac{\delta}{r}\right) + \frac{8a\delta}{pqKr}}\right].$$

Thus (11) is the solution to (*).

Solution - Exercise 6.1

1. The first order condition for maximum of $\pi(e) = pqeX - ce - ae^2 - k - mqeX$ gives

$$(p-m)qX = c - 2ae$$

$$e = \frac{(p-m)qX - c}{2a}$$

2. The optimal harvest is found from $h = qeX$ where e is given above,

$$h = \frac{(p-m)(qX)^2 - cqX}{2a}$$

3. Rearranging the answer to question 2 gives the harvest quota demand function

$$m = p - \frac{c}{qX} - \frac{2a}{(qX)^2}h$$

4. In this case $m = 2500 - 6.25h$

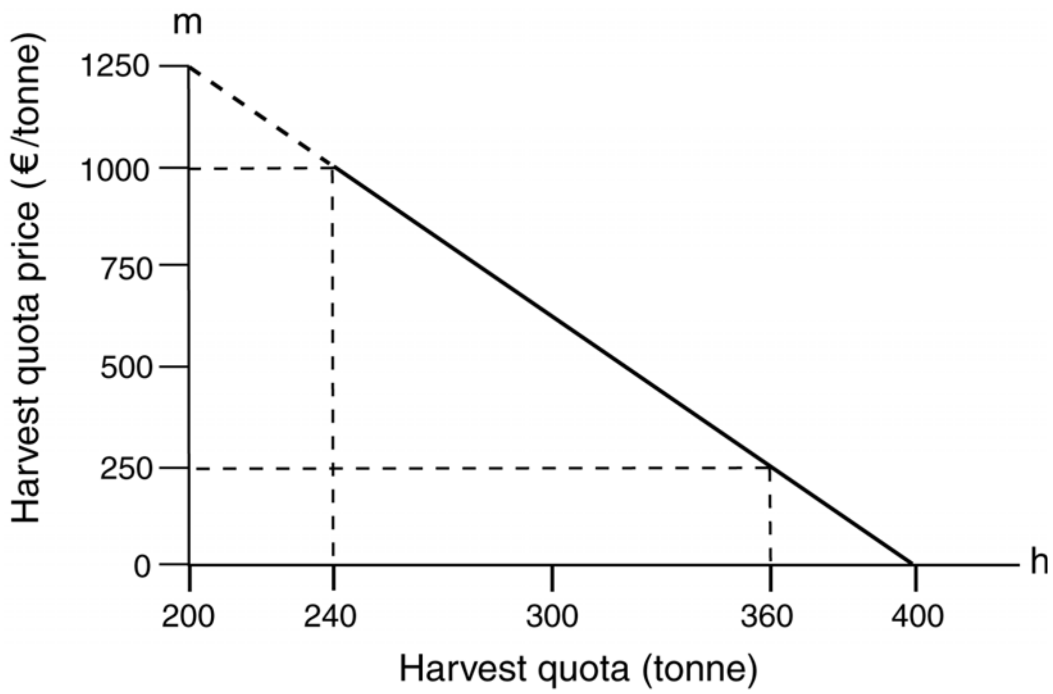


Figure exercise 6.1-1.

5. In this case we have

$$mc(e) = 60 + 0.09e$$

$$avc(e) = 60 + 0.045e$$

$$atc(e) = 60 + 0.045 + 259200/e$$

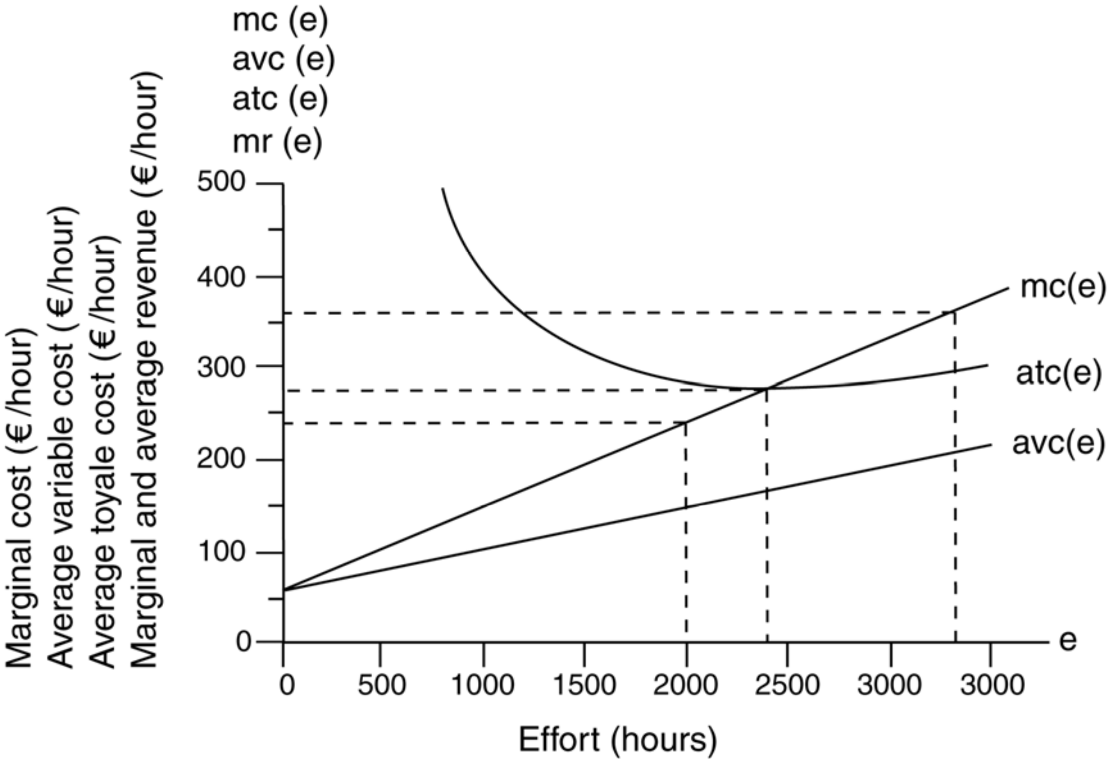


Figure exercise 6.1-2.

Optimal vessel effort is 3333 hours for m = 0 and 2000 hours for m = 1000 €/tonne. The minimum level of average total cost is at e = 2400 hours.

Solution - Exercice 6.2: Fishing vessel economics

$$tc(e) = \frac{1}{3}e^3 - 50e^2 + 2530e + 81\,000$$

a) $mc(e) = e^2 - 100e + 2530$

$$avc(e) = \frac{1}{3}e^2 - 50e + 2530$$

$$atc(e) = \frac{1}{3}e^2 - 50e + 2530 + \frac{81000}{e}$$

b) $mr = pqX = 2055$

Maximum profit for the vessel when:

$mr = mc$

$$2055 = e^2 - 100e + 2530$$

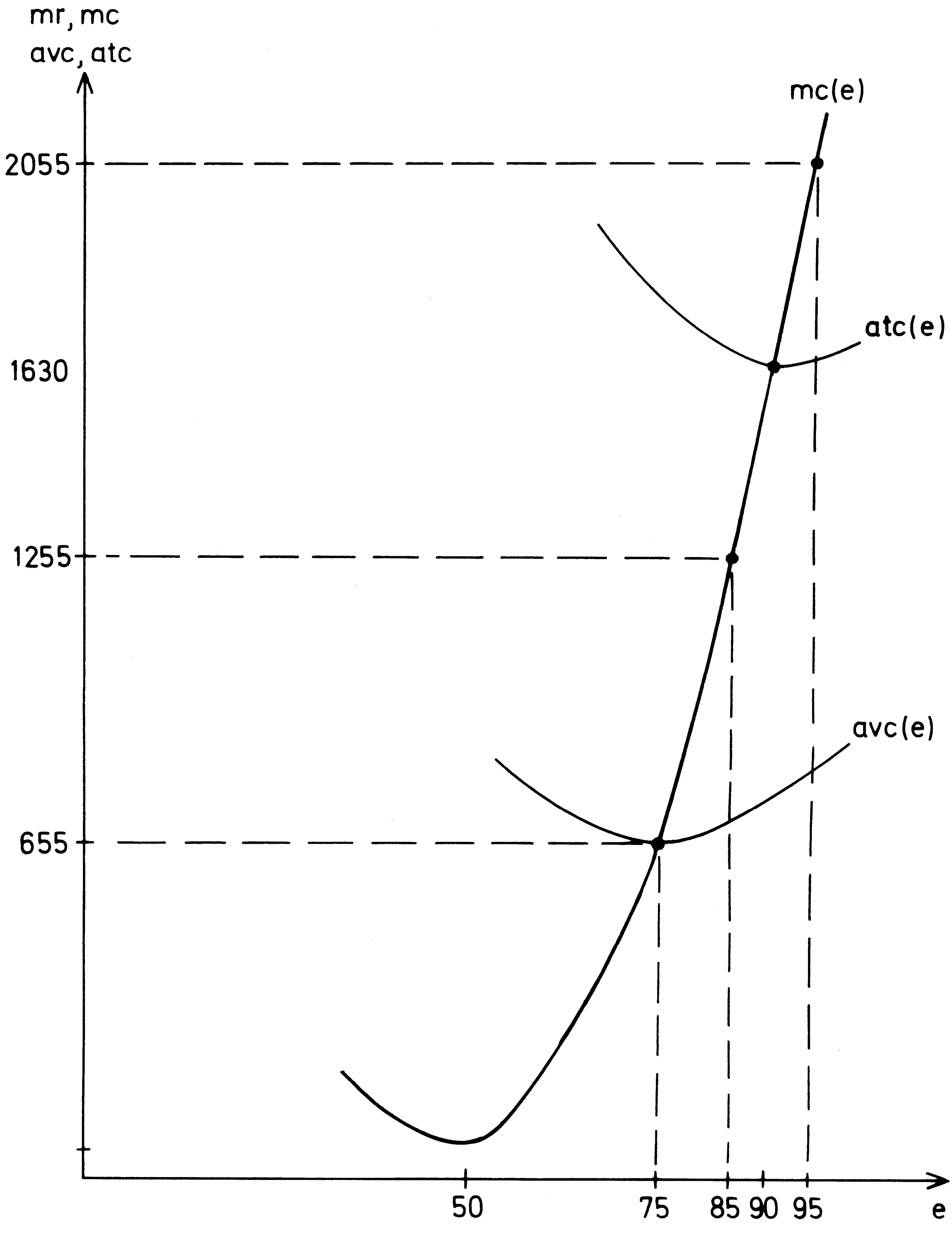
$$e^2 - 100e + 475 = 0$$

$e^* = 95$ (and $e = 5$, but this is a <u>minimum</u> for π)

c) (i) $mr = 1255$, $e^* = 85$

(ii) $mr = 655$, $e^* = 75$

d) Figure



Exercise 8.1.

In a cod fishery fishing mortality is proportional to fishing effort ($F = qE$) and the catchability coefficient is $q = 2.5 \cdot 10^{-4}$ per vessel-year, with unit of time equal to one year. The price of fish is constant (across volume and size of fish), $p = 2.00$ \$/kg, and cost per vessel-year is $a = 0.5$ million \$.

1. What is F when $E = 4000$ vessel-years?

2. Use figure 8.5 to sketch the corresponding graphs of eumetric revenue and total cost of fishing mortality (tip: see figure 8.6 and use cost per unit F, $c = TC/F$, to draw the total cost of fishing mortality curve, $C(F) = cF$).

3. Use the graphs to find approximate values for F_∞^{EU}, F_∞^3, F_∞^0 and F_{MEY}. What are the corresponding number of vessels?

Answer exercise 8.1.

1. $F = qE = 2.5 \cdot 10^{-4} \cdot 4000 = 1.0$.

2. Draw the graph for $R = p \cdot Y^{EU} = 2.00 \cdot Y^{EU}$ \$/kg, by changing the scale on the vertical axis of figure 8.5. Since $TC = aE$ we have cost per unit fishing mortality $c = TC/F = a/q = 2000$ million \$. Thus $C(F) = cF$ depicts a straight line. The graphs should look like the $R^{EU}(F)$ and $C(F)$ graphs in figure 8.6, with \$ and F values along the vertical and horizontal axes, respectively.

3. The values are $F_\infty^{EU} \approx 0.87$, $F_\infty^3 \approx 0.54$, $F_\infty^0 \approx 0.30$, $F_{MEY} \approx 0.22$, $E_\infty^{EU} \approx 3480$, $E_\infty^3 \approx 2160$, $E_\infty^0 \approx 1200$ and $E_{MEY} \approx 880$.

Solution - Exercise 10.1

1. Short run demand:

$$p(D,Q) = \alpha - \beta \cdot D + \gamma \cdot Q$$

Values from table 10.1:

$\alpha = 99.0$, $\beta = 3.125 \cdot 10^{-3}$, $\gamma = 6.25$

$Q_1 = 0.06$:

$$p(D,Q = 0.06) = 99.0 - 0.003125 \cdot D + 0.375$$
$$= 99.375 - 0.003125 \cdot D$$

$Q_2 = 0.15$:

$$p(D,Q = 0.15) = 99.9375 - 0.003125 \cdot D$$

2. Long run demand:

Long run productivity (quality)

$$Q = \frac{H}{D} = q \cdot K\left(1 - \frac{q \cdot D}{r}\right)$$

3. $$p(D) = \alpha - \beta \cdot D + \gamma \cdot qK\left(1 - \frac{q}{r} \cdot D\right)$$

$$p(D) = a - b \cdot D$$

where $a = \alpha + \gamma\, qK$

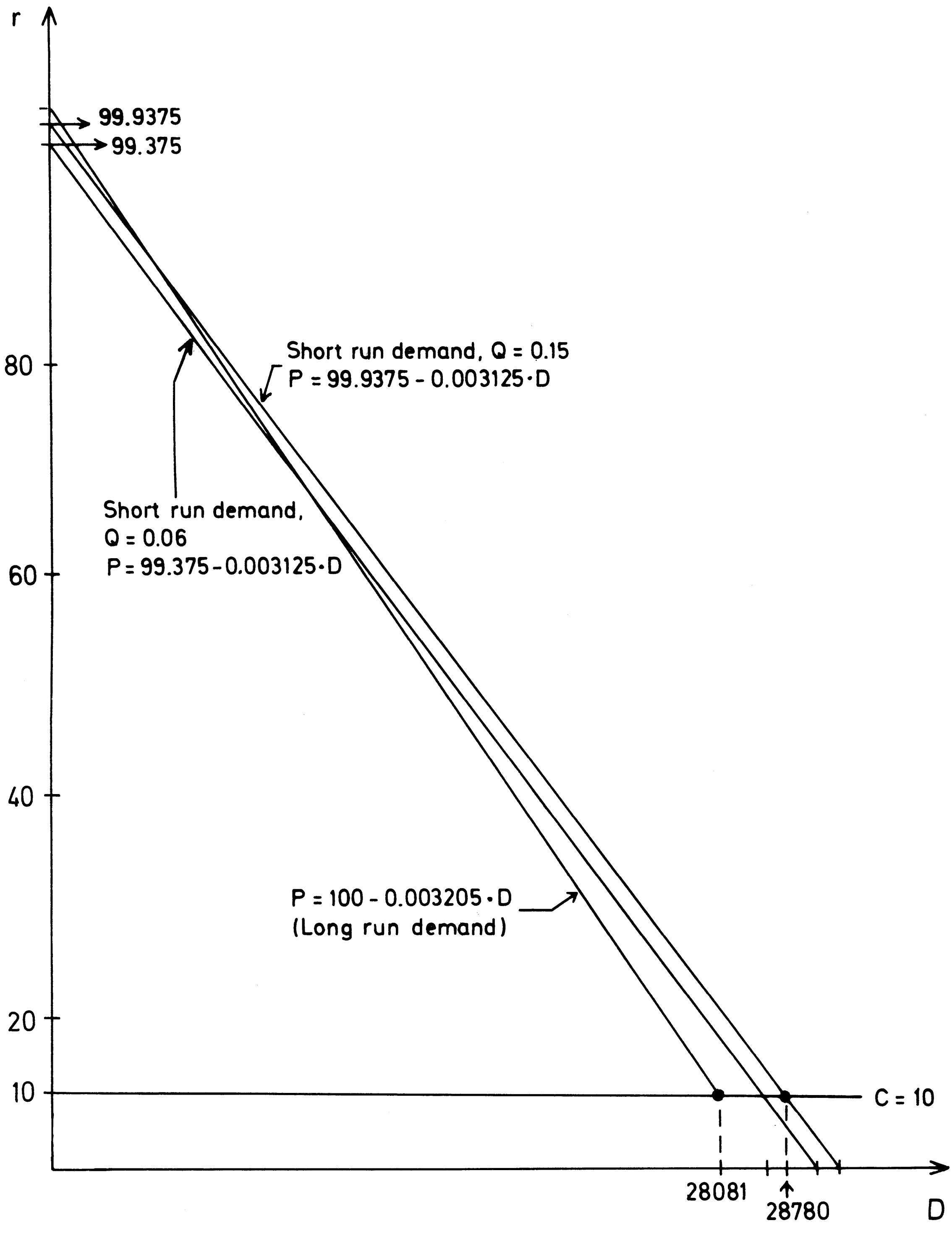
$$b = \beta + \frac{\gamma \cdot q^2 \cdot K}{r}$$

with values from table 10.1

$a = 100$

$b = 0.003205$

Figure for questions 1 and 3



4. The long run demand is a resource adjusted demand which is corrected for the resource effect of angling, which is the negative effect angling has on the stock and on the catch per angler day, Q.

5. Intersection for the short run with the p axis is: $\alpha + \gamma Q$ and the long run is:

$$a = \alpha + \gamma \cdot q \cdot K$$
$$\alpha + \gamma Q = \alpha + \gamma \cdot q \cdot K$$
$$\Rightarrow Q = q \cdot K$$

6. The competitive solution

The short run competitive solution with $Q = 0.15$ is ($p = c$)

$$99.9375 - 0.003125 \cdot D = 10$$
$$\Rightarrow D_S^* = 28790$$

But this is not a sustainable solution.

The competitive angling equilibrium:

$$100 - 0.003205 \cdot D = 10$$
$$\Rightarrow D_L \approx 28081$$

7. The maximum value of the quality indicator is (when $X = K$):

$$Q_{\text{MAX}} = q \cdot K = 0.16$$